

# The Quasar Distribution in a Static Universe

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## ABSTRACT

A crucial test of any cosmological model is the distribution of distant objects such as quasars. Because of well defined selection criteria quasars found by a ultraviolet excess (UVX) survey are ideal candidates for testing the model out to a redshift of  $z = 2.2$ . The static cosmology proposed by Crawford (1993) is used to analyse a recent quasar survey (Boyle et al. 1990). It is shown that the distribution of number of quasars from the survey as a function of redshift is in excellent agreement with the predictions of the model. A  $V/V_m$  test on 351 confirmed quasars with defined redshifts has a mean value of  $0.568 \pm 0.015$  with the discrepancy being most likely due to incompleteness of the catalogue at low redshifts. For the redshift range  $1.5 < z < 2.2$  where the accuracy of the cosmological model is critical  $V/V_m$  was  $0.51 \pm 0.02$ . A well defined quasar luminosity function is derived that has a peak at  $M_B = -21.16$  mag and is well fitted by a Gaussian distribution in absolute magnitude with a standard deviation of 1.52 magnitudes.

*Subject headings:* cosmology: quasars: quasar luminosity function

## 1. Introduction

The cosmological model proposed by Crawford (1993) is stable and static with the Hubble redshift being due to a gravitational interaction (Crawford 1991) that dissipates photon energy into the inter-galactic plasma. Since the model is static the spatial density and any other characteristics of objects must be independent of redshift; that is, there is no possibility of evolution in any form. As a result the model makes simple and specific predictions about how the observed distribution of objects should vary with redshift and magnitude.

Because of their high redshifts quasars are excellent objects for probing the distant universe. Unfortunately most quasar surveys are plagued with selection problems. Most of these have been overcome in a recent survey (Boyle et al. 1990) in which the COSMOS machine was used to find objects with an ultraviolet excess (UVX) using UK Schmidt U and J plates. They obtained spectra for 1400 objects using the Anglo-Australian Telescope. From their spectra 420 objects were identified as quasars (including broad absorption line quasars). Of these 351 that had well defined redshifts with  $z \leq 2.2$  were used as data for this analysis.

In the next section, the relationship between apparent and absolute magnitudes as a function of redshift is derived for the static model. Contrary to the standard (Big-Bang) model (that apart from  $H$ ) there are no free parameters, and thus lack of agreement with the observations would seriously challenge the static model. A description of the observations is given, the  $V/V_m$  statistic is determined and the data are examined for any evidence of evolution. A Monte Carlo calculation is used to show that even with optimum maximum likelihood estimations using accessible volumes there is a remaining bias in the estimated luminosity distribution. Further analysis with allowance for this bias leads to a well defined density distribution of absolute luminosity. The general effects of selection bias are discussed in a simpler context with only one region that has a single magnitude limit and using these results it is argued that the selection bias is important for all cosmological models. The predicted number versus redshift distribution of quasars is computed from the absolute luminosity distribution and it is used to show that the estimation of the luminosity distribution would improved by going to fainter magnitudes.

## 2. The cosmological model

The cosmological model described in Crawford (1993) has the same space geometry as the static Einstein universe (and incidentally the same as a closed standard model that is not expanding) which is that for a three dimensional surface of a four dimensional hyper-sphere. In this theory there is no universal expansion; the Hubble redshift is caused by a gravitational interaction with the inter-galactic plasma (Crawford 1987). The interaction cause the photon to lose energy via the emission of very many very low energy secondary photons. In most cases the frequency of the secondaries is well below the plasma frequency in which case the energy loss is via direct transfer to plasma waves. In either case the rate of energy loss is dependent on the local plasma density. The accumulated energy loss causes a redshift that is a function of the distance and the density of the plasma. Provided the plasma density does not vary significantly the redshift is a good measure of the distance to an object. This is certainly true for large scales. For smaller scales such as the size of galactic clusters there will be variations in the plasma density that are reflected as variations in the redshift. In this theory the voids and walls seen in the distribution of galaxies are primarily due to variations in the plasma density. However these variations are small compared to the quasar redshift distances.

Since the cosmological model is static and is not evolving it obeys the *perfect cosmological principle* (Bondi & Gold 1948) in which there is both spatial and time isotropy. This does not prevent objects such as galaxies or quasars from evolving. What it does require is that their creation and evolution is independent of their location in space and in time. Hence any sampling of the universe over a sufficiently large scale should not detect any variation in the average characteristics of the objects such as luminosity or density. The application of this principle to quasars requires that their spatial density and luminosity should be independent of their redshift. A critical test that would refute the static cosmology is unequivocal evidence of evolution of luminosity or density of objects as a function of redshift. Whereas the standard model not only permits evolution but since in it galaxies have a finite lifetime and a common birth time, it demands evolution of galaxies and presumably quasars. Lack of observed evolution in the standard model can only arise by a fortuitous coincidence between the effects of expansion and galactic evolution.

If  $R$  is the radius of the hyper-sphere that describes the geometry of this static cosmology then the two-dimensional surface area of the three-dimensional sphere of radius  $r$  is

$$A(r) = 4\pi R^2 \sin^2(r/R), \quad (1)$$

and the three-dimensional volume is

$$V(r) = 2\pi R^2 \left( r - \frac{R}{2} \sin\left(\frac{2r}{R}\right) \right), \quad (2)$$

where  $r$  can vary from 0 to  $\pi R$ . Thus the total volume of the universe on this model is  $2\pi^2 R^3$ .

From Crawford (1991) the redshift ( $z = \lambda_0/\lambda_e - 1$ ) for a photon that has travelled a distance  $r$  is given by

$$z = \exp\left(\frac{Hr}{c}\right) - 1, \quad (3)$$

where  $H$  is the Hubble constant. One of the major results of the model is to relate the Hubble constant to the universe's radius  $R$ . From Crawford (1993) we get  $R = \sqrt{2}c/H$  which provides the basic relationship

$$r = \frac{R}{\sqrt{2}} \ln(1 + z). \quad (4)$$

Since the maximum value of  $r$  is  $\pi R$  equation (4) shows that the maximum value of  $z$  is 84.02 and that  $z = 8.22$  when  $r = \pi R/2$ .

Suppose there are quasars (or any other objects) with spatial density  $\rho$  then the number in the interval  $z$  to  $z + dz$  is given by

$$n(z) = \rho \frac{dV}{dz} = \rho \frac{dV}{dr} \frac{dr}{dz} = \frac{2\sqrt{2}\pi R^3 \rho \sin^2(\ln(1+z)/\sqrt{2})}{1+z}. \quad (5)$$

This distribution has a maximum value when  $z = 2.861$ .

Let a source of radiation have a luminosity  $L(\nu)$  (W.Hz<sup>-1</sup>) at the emission frequency  $\nu$ . Then if energy is conserved the observed flux density  $S(\nu_0)$  (W.m<sup>-2</sup>.Hz<sup>-1</sup>) at a distance  $r$  is the luminosity divided by the area (equation (1)) which is

$$S(\nu_0) = \frac{L(\nu)}{4\pi R^2 \sin^2(r/R)}.$$

However because of the gravitational interaction there is an energy loss such that the received frequency  $\nu_0$  is related to the emitted frequency  $\nu_e$  by equation (3) and is

$$\nu_0 = \nu_e \exp(\sqrt{2}r/R) = \nu_e/(1+z).$$

The loss in energy means that the observed flux density is decreased by a factor of  $1+z$ . But there is an additional bandwidth factor  $d\nu_e = (1+z)\nu_0$  that tends to balance the energy loss factor. The balance is not perfect because the source is observed at a different part of its spectrum from that for a similar nearby source. The correction for this spectral offset is called the K-correction and Rowan-Robinson (1985) defines it by

$$K(z) = -2.5 \log \left( \frac{(1+z) \int \psi(\nu_0) L((1+z)\nu_0) d\nu_0}{\int \psi(\nu_0) L(\nu_0) d\nu_0} \right)$$

where  $\psi(\nu)$  is the filter transfer function and  $K$  is expressed in magnitude units. Note that the bandwidth correction is explicitly included in the  $K$ -correction. Then the apparent magnitude  $m$  is given by

$$\begin{aligned} m &= -2.5 \log(S(\nu_0)) \\ &= -2.5 \log(L(\nu_0)) + 5 \log(\sin(r/R)) \\ &\quad -2.5 \log(4\pi R^2) + 2.5 \log(1+z) + K(z) \end{aligned}$$

Since the absolute magnitude  $M$  is equal to the apparent magnitude at a distance of  $10pc$  and using the relationship between  $R$  and  $H$  we get

$$M = -2.5 \log(L(\nu_0)) - 43.761 - 2.5 \log(4\pi R^2)$$

where it is assumed that  $H = 75 \text{ km.s}^{-1}.\text{Mpc}^{-1}$ .

Hence the apparent magnitude is

$$m = M + 5 \log(\sin(\ln(1+z)/\sqrt{2})) + 2.5 \log(1+z) + K(z) + 43.761. \quad (6)$$

A common assumption (Boyle et al. 1990) is that on average the continuum spectrum for quasars is of the form  $L \propto \nu^\alpha$ . With this power law spectrum the  $K$ -correction is

$$K(z) = -2.5(1+\alpha) \log(1+z).$$

Then the magnitude relation for a power law spectrum is

$$m = M + 5 \log(\sin(\ln(1+z)/\sqrt{2})) - 2.5\alpha \log(1+z) + 43.761. \quad (7)$$

Let the density distribution of sources as a function of absolute magnitude be  $f(M)$  such that the density of sources in the range  $M$  to  $M + dM$  is  $d\rho = \rho_0 f(M) dM$  where  $f(M)$  is normalized to one. Then for  $z$  in the range  $z$  to  $z + dz$  the and using equation (5) the observed number of sources with apparent magnitudes in the range  $m$  to  $m + dm$  is

$$n(m)dm = 2\sqrt{2}\pi\rho_0 R^3 \int_0^{84.02} dz \frac{f(M(m,z)) \sin^2(\ln(1+z)/\sqrt{2})}{1+z} dm \quad (8)$$

where equation (6) or equation (7) is used to calculate the absolute magnitude. Thus the absolute luminosity distribution and the  $K$ -corrections must be known before the apparent magnitude distribution can be calculated.

### 3. The observations

As mentioned the set of observations used in this study is the catalogue of quasars described by Boyle et al (1990). They used the COSMOS machine to measure the apparent magnitudes on 8 sets of U and J plates taken with the UK Schmidt telescope. In this paper all the B magnitudes are their uncorrected values and should be reduced by  $0.1 \pm 0.05$  mag to get Johnston magnitudes. The U magnitudes were only used to get the  $U-B$  difference for object selection. Objects were selected for further spectral analysis using the FOCAP system on the Anglo-Australian Telescope. This is a multiple optical fibre spectrometer that could measure the spectra of up to 64 objects in a  $0.35 \text{ deg}^2$  field. There were 34 fields spread over the 8 pairs of plates. To be included an object had to have an ultra-violet excess which meant that  $U-B$  was less than a value that was field dependent and which varied from  $-0.05$  to  $-0.65$  mag. Because of seeing and other instrumental effects each field had its own magnitude limits, typically from about 17 to 21 mag. After analysing the spectra they found that 420 objects were quasars (including 9 broad emission line objects) and most of the remainder were halo subdwarfs. The current analysis is restricted to a subset of 351 quasars each of which had a well defined redshift.

Examination of the individual spectra provided by Boyle et al (1990) shows that there is wide variation from a constant spectral index of  $\alpha = -0.5$ . The major requirement for the K-correction is in determining the B magnitude at the emission wavelengths (near  $4300\text{\AA}$ ) from a value which has been observed at longer wavelengths. Since the spectra covered the range from  $3600\text{\AA}$  to  $6600\text{\AA}$  they could be used to get an estimate of the K-correction. To do this it was assumed that the spectra could be approximated by a power law and a set of templates were used to estimate the power law index from the published spectra. The spectral indices estimated from the templates had a mean of  $-0.59$  and a standard deviation of  $0.73$ . Although crude it is slightly better than assuming a constant spectral index.

In order to calculate the accessible volume the K-correction is also needed for wavelengths shorter than the B band. In this case the  $U-B$  color index can be used to get a rough estimate of the spectral index. By integrating the U and B response curves (Johnson & Morgan 1953) multiplied by power laws it was found that a good estimate of the power law index is given by  $\alpha = -3.32 - 4.0 (U-B)$  where the constant term is chosen so that when  $U-B = -1.33$  mag the spectral index is 2.0. In addition the spectral indices estimated from the  $U-B$  color were restricted to lie in the range  $-2 \leq \alpha \leq 2$ . These spectral indices had a mean of  $-0.32$  and a standard deviation of  $1.27$  and for the selected quasars the correlation coefficient between the two indices was  $0.7$ . Thus the K-correction was estimated assuming a power law spectrum with the index determined

from template matching to the published spectra for corrections to lower values of  $z$ , and using the  $U-B$  spectral index for corrections to higher values of  $z$ .

Boyle et al (1987) state that errors on the calibration of B magnitudes range from  $\pm 0.10$  mag at B = 18.0 mag to  $\pm 0.15$  mag at B = 21.0 mag, with errors in  $U-B$  colors in the range  $\pm 0.15$  mag to  $\pm 0.25$  mag. Of greater importance is the completeness of the catalogue which is discussed extensively in Boyle et al (1987). They claim a completeness of greater than 80% for quasars with  $0.5 \leq z \leq 0.9$  and for the whole catalogue a completeness of 90%. Following their lead the analysis described here was restricted to the range  $0.3 \leq z \leq 2.2$  which reduced the number of quasars to 351.

#### 4. The $V/V_m$ test

The  $V/V_m$  test introduced by Schmidt (1968) and Rowan-Robinson (1968) is the average for all sources of the ratio of the actual volume between the earth and the source divided by the volume between the earth and the furthest point at which the source would no longer satisfies the selection criteria. It is a statistic from the uniform distribution (0,1) with a mean of 0.5 and a standard deviation for  $n$  sources of  $1/\sqrt{12n}$ . For the data from Boyle et al (1990) there are bright magnitude limits and the appropriate statistic is  $U/U_m$  (Avni & Bachall 1980) which has the same characteristics as  $V/V_m$ . If the  $i$ 'th quasar has a radial distance  $r_i$  and its accessible volume lies between the radii  $a_i$  and  $b_i$  then the  $U/U_m$  statistic is defined by

$$\left\langle \frac{U}{U_m} \right\rangle = \frac{1}{n} \sum_{i=1}^n \frac{v(r_i) - v(a_i)}{v(b_i) - v(a_i)}.$$

where  $v(r_i)$  is the volume defined by

$$v(r_i) = \sum_j A_j V(r_{ij}), \tag{9}$$

and where the index  $j$  runs over those regions in which the quasar could have been observed,  $A_j$  is the solid angle of each region and the volume  $V(r_{ij})$  is computed using equation (2). The  $v(a_i)$  and  $v(b_i)$  are computed using equation (9) except that the radii used are the minimum or maximum radii (respectively) for which the quasar could have been observed in each region. The absolute magnitude for each quasar was computed using equation (7). For the 351 selected quasars 198 had their maximum volume determined by the  $z$ -limit. That is they were bright enough to be seen at  $z = 2.2$  and larger distances.



For 351 quasars the observed mean value is  $\langle U/U_m \rangle = 0.569 \pm 0.015$  which is not in statistical agreement with the expected value of 0.5. The observed standard deviation of 0.253 is in reasonable agreement with the expected value of 0.289. However the number of quasars in each decile of  $\langle U/U_m \rangle$  which is given in table (1) shows that the discrepancy is mainly due to a deficiency of quasars in the first two deciles. There appears to be a deficiency of about 50 weak nearby quasars, or 14% of the total which is consistent with the limits of completeness of the catalogue. As a further check the data was divided into two groups, 191 sources with  $0.3 < z < 1.5$  and 160 sources with  $1.5 < z \leq 2.2$ . For the first group  $\langle U/U_m \rangle = 0.58 \pm 0.02$  and for the second group it was  $0.51 \pm 0.02$ . Since the differences between cosmological models are negligible for low values of redshift one would expect that if the model was incorrect the higher redshift group (with  $\langle U/U_m \rangle = 0.51$ ) would show the largest deviation from 0.5. The fact that it is the low redshift group that shows the major discrepancy supports the argument that the difference is mainly due to selection effects.

In order to show that the  $\langle U/U_m \rangle$  ratios were not strongly dependent on the K-corrections that were used the analysis was repeated with a constant K-correction of  $-0.5$ . For the range  $0.3 < z < 2.2$  the result was 0.574 to be compared with 0.569 (above)R, and for the same two groups the values were identical to those with a variable K-correction.

Also shown in table (1) are the average redshift, average apparent magnitude, the average absolute magnitude and the average value of  $U-B$  for each decile. As a function of  $U/U_m$  there is a large (as expected) dependence of  $z$  and a small, but hardly significant (probability by chance of  $\sim 5\%$ ), variation in absolute magnitude and no apparent variation in  $U-B$ . The expected result for a static cosmology is that any characteristic of the quasars should be independent of  $U/U_m$ . Overall the results show that apart from small, possibly selection, effects, the static cosmology is consistent with this data on the  $U/U_m$  test.

## 5. The quasar luminosity function

The accessible volume for each quasar ( $V_i$ ) is defined using equation (9) by

$$V_i = v(b_i) - v(a_i).$$

The luminosity distribution is obtained by selecting those quasars within a small absolute magnitude range and dividing the number of quasars by the average of their accessible volumes. For Poissonian statistics this is the maximum likelihood estimate.

Because of the strong selection effects on magnitude and redshift the data only span part of the quasar luminosity range and as a result the direct determination of the luminosity function is biased. In order to determine this selection bias a Monte Carlo program was used to simulate the selection effects. The procedure was to choose a random absolute luminosity from a given luminosity function and a random position within the maximum accessible volume. Next a spectral index from the uniform distribution  $(-2.0, 2.0)$  was chosen and the  $U-B$  index was calculated from  $U-B = -0.25\alpha - 0.83$ . The apparent magnitude and redshift were then calculated using equations (3) and (7). It was assumed that there is no correlation between absolute magnitude and spectral index. If this hypothetical object satisfied the selection limits for one of the 34 areas it was accepted for further processing. Finally in order to simulate the deficiency of weak sources the  $V/V_m$  ratio was calculated and if it was in the first decile 75% of the objects were rejected and if it was in the second decile 50% were rejected. The simulated objects were then analysed by the same method used to analyse the quasars. The main purpose of this analysis was to get density correction factors as a function of absolute magnitude. Although the accessible volume method corrects for most of the selection effects the Monte Carlo analysis showed that correction factors were still required to allow for the a priori distribution of the quasar luminosities. For example if the parent luminosity distribution was a Gaussian in absolute magnitude with a mean of  $M_* = -21.15$  mag and with a standard deviation in the range  $1.0 \leq \sigma_0 \leq 2.5$  mag then the observed luminosity distribution is closely approximated by a Gaussian with a standard deviation  $\sigma = 0.27 + 0.73\sigma_0$  and with an observed mean varying from  $-22.33$  mag to  $-22.92$  mag. In effect the selection effects truncate the distribution giving smaller standard deviations.

Correction factors were calculated as a function of absolute magnitude (in half magnitude steps) using a luminosity distribution that is a Gaussian in absolute magnitude with parameters  $(-22.16 \text{ mag}, 1.52 \text{ mag})$ . It should be noted that the correction factors are only weakly dependent of the nature of this parent distribution. In fact the parent distribution is only required to determine the correct weighting within each magnitude box. However the factors are strongly dependent on the magnitude and redshift selections. The correction factors (that multiply the observed densities) for  $H = 75 \text{ km.s}^{-1}.\text{Mpc}^{-1}$  and for three redshift ranges are shown in table 2.

The results for the density distribution (using the correction factors) of the quasars as a function of absolute magnitude is shown in figure (1). The densities have been increased by a factor of 416/351 (4 of the 420 quasars had  $z > 2.2$ ) to allow for quasars that were not included.

Table (3) gives the magnitude range, the number of quasars, the quasar density and

the mean value of  $U-B$  for each of the magnitude ranges. The curve in figure (1) is the best fit Gaussian (chosen for analytic simplicity) with the form

$$f(M) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\left(\frac{M - M_*}{\sigma}\right)^2\right) \quad (10)$$

where  $M$  is the blue magnitude and since  $f(M)$  is normalized to one it must be multiplied by the total quasar density  $\rho_0$  to get densities. The values for the parameters for three values of the Hubble constant are shown in table (4) where quoted uncertainties are estimated standard deviations derived from the Monte Carlos simulations. These values were derived from independent runs using different values for the Hubble constant and fully include the effects of non-linearities in the equations.

What is most remarkable about this luminosity distribution is that it has a well defined peak. Whereas luminosity functions derived using the standard model (Hartwick & Schade 1990, Boyle et al. 1987) rarely show a peak and are spread over a much larger range in magnitudes. Furthermore they are different for different redshift ranges and there is little agreement on the type of evolution that is needed to give consistent results. In contrast the static cosmology used here has (apart from  $H$ ) no free parameters and produces a well defined luminosity function. Since the observed luminosity function will be broader than the actual luminosity function because of uncertainties in the magnitudes, uncertainties in the K-correction and because of intrinsic variability in the magnitudes of quasars (Hawkins & Vèron 1993), the narrowness of the observed luminosity function suggests that the contribution to its width from the errors in the cosmological model is small and thus there is strong confirmation for the validity of the static cosmological model. As a further check the analysis was repeated with a constant K-correction of  $-0.5$  with the result that the luminosity function was still peaked but with a much larger width which shows that the use of the computed K-corrections provides a worthwhile improvement.

In order to see if there are any evolutionary effects the analysis for the luminosity distribution for three redshift ranges with the same magnitude limits are shown (for  $H = 75 \text{ km.s}^{-1}.\text{Mpc}^{-1}$ ) in table (5) where the uncertainties ( $1\sigma$ ) have been estimated from repeated Monte Carlo simulations. A statistical  $t$  test shows that the differences in absolute density between the near group and the far group is not significant at the 5% level and the differences in peak magnitude is not significant at the 2.5% level. Even if we admit a difference in magnitude it could be easily due to the different effective magnitude ranges or due to uncertainties in the K-correction which has little effect on the nearby quasars and much larger effects on the distant quasars. Although there may be some effect due to deficiencies in the cosmological model the reduction in the width of the peak for the two groups is mainly due to the restricted magnitude range in each case. The overall result is

that there is no obvious evidence for evolution and that the static cosmological model is consistent with the observations.

## 6. Discussion

### 6.1. Selection bias

The Monte Carlo results show that a strong selection bias still exists even with maximum likelihood estimation of densities. The cause of this bias is the non-linear relationship between accessible volume, the absolute magnitude and the selected apparent magnitude ranges. There are lesser dependencies on the shape of the luminosity function and color selection (via the K-correction). Figure (2) shows the correction factor as a function of magnitude for a simple example that has one region with  $0.0 \leq z \leq 2.2$  and a single magnitude limit (with  $H = 75 \text{ km.s}^{-1}.\text{Mpc}^{-1}$  and variable spectral index as described above). Each curve is the Monte Carlo results for a different magnitude cut-off. Although these results are computed for the static cosmological model it is clear that the bias will exist for any cosmological model. Repeated trials show that although the complex behaviour seen on the left for the brighter limits ( $B = 19 \text{ mag}$  and  $B = 20 \text{ mag}$ ) is genuine it should not be taken too seriously since the observed number of sources in this region is very low. The brightest peak in each curve corresponds to the absolute magnitude of an object with the limiting apparent magnitude at a redshift of about  $z = 0.36$ . Because of small numbers the bias to the left of the brightest peak is very poorly determined. Consider the curve with a cut-off at  $B = 20 \text{ mag}$ . It has a peak correction of 3.5 at  $M_B = -20.8 \text{ mag}$ . However in a survey only about 3% of the objects (within the full redshift range) fainter than this would be detected. It is because of the high rejection rate that the non-linear selection effects become important. Again it should be emphasized that although the details of the bias are dependent on the cosmological model the analysis of any survey with any cosmological model should allow for selection bias.

### 6.2. The redshift distribution

Given the luminosity distribution it is a straight forward integration using equation (8) to derive the number of quasars that could be seen brighter than a magnitude limit as a function

of redshift. Figure (3) shows the expected number of quasars as a function of redshift for seven magnitude limits. The adopted luminosity distribution is a Gaussian in absolute magnitude ( $-21.16$  mag,  $1.52$  mag) and the curves are calculated for  $H = 75 \text{ km.s}^{-1}.\text{Mpc}^{-1}$ . As the magnitude limit is increased the curves peak at larger redshifts and asymptotically approach equation (5). Note that these are theoretical curves and do not include the effects of selection bias and therefore they cannot be directly compared with observational data.

### 6.3. Further studies

The success of the static cosmological model in producing a well defined luminosity distribution enables one to use the model to see what improvements could be made to future quasar surveys in order to achieve a better luminosity function. Clearly the major difficulty with current surveys is the limited magnitude range. Figure (3) shows that at  $z = 2.0$  only about 36% of the quasars are observed with a magnitude less than  $B = 21$  and we have to go to  $B = 24$  to get 94% completeness. Thus it is clear that determination of the luminosity distribution requires fainter magnitude limits with well defined selection criteria. Whereas tests of cosmological theories benefit more from larger redshift ranges. A second improvement would be better estimation of the K-correction. At large redshifts uncertainties in it easily dominate the uncertainties in the absolute magnitude. The ideal would be accurate integrations of each quasar spectrum at zero redshift, the observed redshift and at the survey limiting redshift. Of further interest is that the Monte Carlo simulations show that with larger numbers and a larger redshift range there is the possibility of determining  $H$  using quasar catalogues. This is because the non-linearities in the magnitude-redshift relationship and the volume-redshift relationship are becoming important.

## 7. Conclusion

It has been shown that the model of a static universe is consistent with the observed quasar distributions. Subject to the uncertainty associated with the unknown K-correction the  $\langle U/U_m \rangle = 0.568 \pm 0.015$ , and the results given in table (1) shows that apart from a lack of weak nearby quasars there is no reason to reject the static cosmological model with the  $U/U_m$  test, in fact there is strong support. The fact that the absolute magnitude density distribution shown in figure(1) has a well defined peak is strong evidence for the validity

of the static model in explaining the density distribution and magnitude relationship for quasars. If the theory was seriously in error the peak would be very broad or non existent. Since this agreement of the static model with the data is achieved without the fitting of any free parameters it is strong support for the validity of the theory. Furthermore it is apparent from this analysis that the major deficiency in this data that limits its use as test for cosmological theories is its limited range in magnitudes and uncertainties in the K-corrections.

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Table 1:  $U/U_M$  distributions

decile	number	$\langle z \rangle$	$\langle B \rangle$	$\langle M_B \rangle$	$\langle U-B \rangle$
1	10	0.556	18.74	−22.29	−0.74
2	20	0.707	19.38	−22.55	−0.74
3	35	0.950	19.62	−23.02	−0.83
4	37	1.128	19.80	−23.08	−0.86
5	39	1.224	19.98	−22.95	−0.75
6	41	1.406	19.84	−23.07	−0.68
7	45	1.514	20.24	−22.87	−0.77
8	49	1.671	20.27	−23.08	−0.80
9	38	1.806	20.30	−23.02	−0.59
10	37	1.988	20.29	−23.45	−0.70

Table 2: Density correction factors

$\langle M_B \rangle$	$0.3 \leq z \leq 2.2$	$0.3 \leq z \leq 1.5$	$1.5 \leq z \leq 2.2$
–19.25	1.479	0.928	
–19.75	1.218	0.885	2.689
–20.25	1.075	0.803	1.809
–20.75	0.879	0.738	1.372
–21.25	0.788	0.692	1.090
–21.75	0.706	0.663	0.912
–22.25	0.656	0.643	0.786
–22.75	0.612	0.616	0.660
–23.25	0.564	0.577	0.560
–23.75	0.526	0.513	0.488
–24.25	0.480	0.532	0.441
–24.75	0.515	0.564	0.481
–25.25	0.567	0.572	0.531
–25.75	0.608	0.591	0.571
–26.25	0.624	0.586	0.617



Table 3: Quaser luminosity function

$\langle M_B \rangle$	count	$\rho_0 f / (\text{Gpc}^{-3} \cdot \text{mag}^{-1})$	$\langle U - B \rangle$
−19.85	1	$268 \pm 268$	−0.30
−20.30	5	$475 \pm 277$	−0.72
−20.75	15	$895 \pm 291$	−0.73
−21.27	23	$1429 \pm 324$	−0.69
−21.76	23	$968 \pm 241$	−0.61
−22.27	45	$1396 \pm 141$	−0.80
−22.77	58	$1216 \pm 170$	−0.77
−23.27	63	$1027 \pm 134$	−0.79
−23.74	47	$679 \pm 102$	−0.75
−24.26	36	$471 \pm 80$	−0.81
−24.75	18	$261 \pm 63$	−0.74
−25.25	14	$249 \pm 68$	−0.59
−25.85	2	$36 \pm 26$	−0.73
−26.25	1	$26 \pm 26$	−0.50

Table 4: Luminosity parameters for different Hubble constants

$H/\text{km.s}^{-1}.\text{Mpc}^{-1}$	$\rho_0/\text{Gpc}^3$	$M_*$	$\sigma$
50	$1180 \pm 58$	$-22.94 \pm 0.12$	$1.58 \pm 0.12$
75	$4820 \pm 240$	$-22.16 \pm 0.12$	$1.52 \pm 0.12$
100	$13400 \pm 660$	$-21.39 \pm 0.12$	$1.62 \pm 0.12$

Table 5: Luminosity parameters for different redshift selections

selection	$\rho_0/\text{Gpc}^3$	$M_*$	$\sigma$
$0.0 \leq z \leq 2.2$	$4820 \pm 240$	$-22.16 \pm 0.12$	$1.52 \pm 0.12$
$0.0 \leq z \leq 1.5$	$4570 \pm 510$	$-22.06 \pm 0.22$	$1.21 \pm 0.23$
$1.5 \leq z \leq 2.2$	$3020 \pm 530$	$-23.10 \pm 0.33$	$1.33 \pm 0.27$

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Fig. 1.— Luminosity function for UVX quasars as a function of absolute magnitude. The unit is number per  $\text{Gpc}^3$  per unit magnitude interval. The curve is the best fit Gaussian (see text)

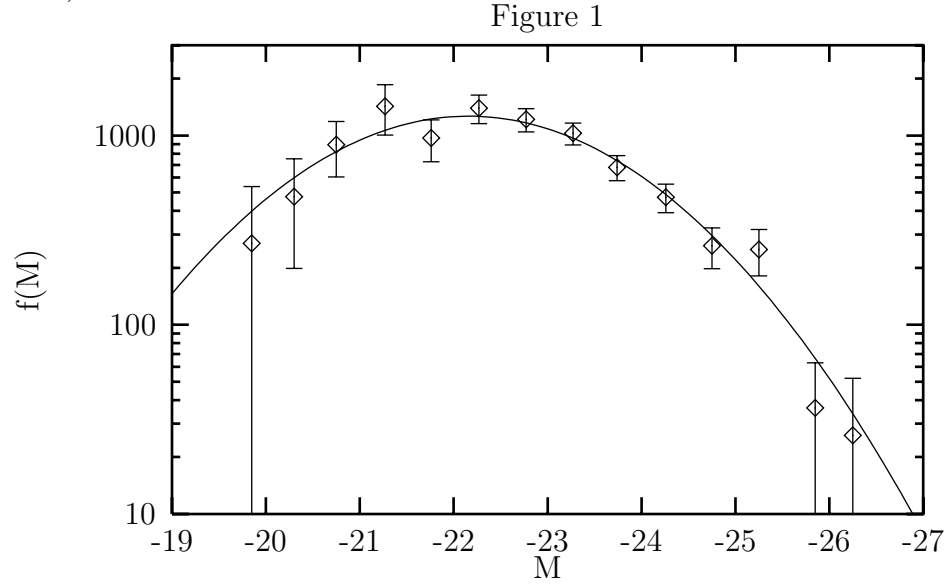


Fig. 2.— Selection bias factor for a single region for magnitude limits of  $B=19,20,21,22,23$  and 24.

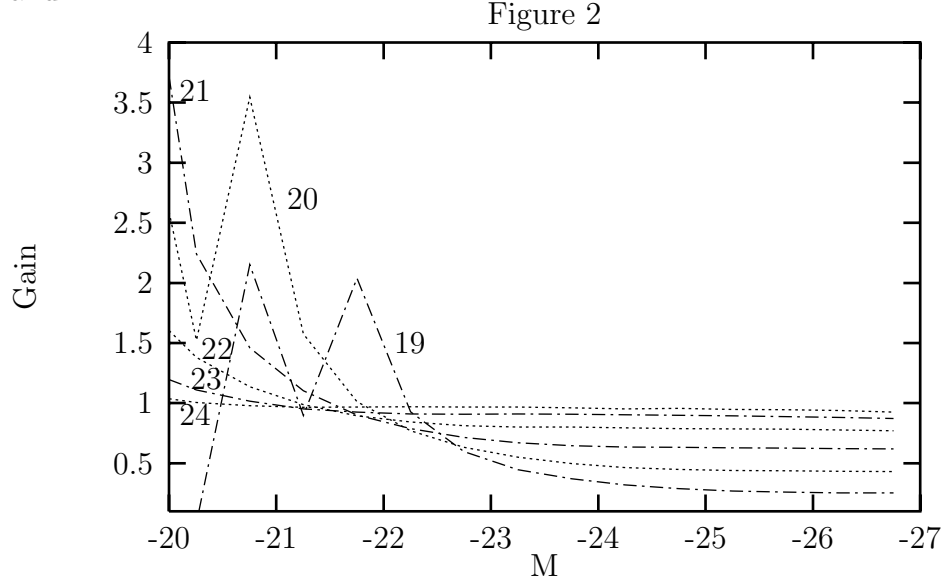


Fig. 3.— Number of quasars per square degree as a function of redshift for magnitude limits of  $B=19,20,21,22,23,24$  and 25.

